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A METHOD

OF CORRECTING THE APPROXIMATE ELEMENTS OF THE *ORBIT OF A COMET*,

AND THE APPLICATION OF THE SAME TO OBSERVATIONS MADE AT THE OBSERVATORY OF *TRINITY COLLEGE DUBLIN*, ON THE COMET OF JULY 1819.

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WHATEVER difference of opinion there may be among astronomers, relative to the respective convenience of the different methods of obtaining approximate elements of the orbit of a Comet, there can be none, I think, relative to the superiority of M. Laplace's method of correcting the approximate elements when found. But this method may, as it appears to me, be considerably improved in the following manner.

The method as given in the "Mecanique Celeste,"* may sometimes become impracticable, or at least, lead to tedious computations for want of a knowledge of the degree of exactness required in the approximate elements. M Delambre, after describing † M. Laplace's method, remarks, "Cette methode est fort belle; si elle est peu longue, elle réussit presque toujours; elle n'est pourtant pas infaillible, et aprés de très longs calculs, on a été forcé d'y renoncer pour la seconde comète de 1813."

Not being able to refer to the observations used for that Comet, it is not certain whence the difficulty arose, whether in obtaining the approximate elements or in the correction. It is most likely in the latter; and, if so, the method here proposed would have prevented the very long calculations, and probably would have removed the whole difficulty.

M. Laplace's method of correction is as follows:

He takes three observations of the Comet, separated from each other by considerable intervals; and, with the approximate perihelion distance (p) and approximate time of perihelion, he computes three anomalies, (ν, ν', ν'') and three radii vectores (r, r', r'') Let $\nu'-\nu=U$, $\nu''-\nu=U'$. Let also β, β', β'' and π, π', π'' be the respective heliocentric longitudes and latitudes. He computes these by means of the observed geocentric longitudes and latitudes, the Comet's distances from the sun, and the earth's distances from the sun. From the heliocentric longitudes and latitudes he computes the angles at the sun, subtended between the first and second places of the Comet, and between the first and third. These angles he calls V and V' respectively. Then, if the approximate elements were correct,

$$V=U \text{ and } V'=U'.$$

This not being so, he supposes $m = U-V$ and $m'=U'-V'$.

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* Tom. 1. p. 225 &c.

† Delambre, Astr. Tom. 3. p. 386.

Then 1. Changing only the approximate perihelion distance by a small quantity, he obtains new values of m and m'.

2. Changing only the time of perihelion by a small quantity, he obtains other values of m and m'.

From the relation of these values, he obtains two equations, by the solution of which the corrections of the perihelion distance, and of the time of perihelion are obtained.

If these be not sufficiently correct, as will generally be the case, new corrections must be investigated.

Every correction of the elements requires nine long operations, in which, to obtain the necessary exactness, it is requisite to use logarithms of seven places of decimals.

By the alteration I am about to suggest, there will be required for the first corrections only three operations, in which seven places of figures will be necessary; in the other part of the process, five or even four places of logarithms will be sufficient; and, in the subsequent corrections, only three operations to seven places of figures will be necessary. The repetition of the part of the process in which four or five places only were used, may in general be dispensed with.

Instead of two hypothesis as in M. Laplace's method, by which two equations with two unknown quantities are obtained, I obtain without any hypothesis, two equations in which the fluxion of the perihelion distance and of the time from perihelion are the unknown quantities, in the following manner:—

Let the true values of U and V be $U + \delta U$ and $V + \delta V$.

Then $U + \delta U = V + \delta V$ and $U' + \delta U' = V' + \delta V'$.

Now we can compute δU, $\delta U'$, δV, $\delta V'$ so as to obtain two equations of the form

$$a dp + b dt \quad V - U$$
$$a' dp + b' dt' = V' - U'$$

From which the values of dp and dt may be obtained, t being the time from perihelion.

Let T = the time in the table of the Comet of 109 days when anomaly $= \nu$

Δ = change of anomaly in that Table in one day at time T

$$\cos^2 \tfrac{1}{2}\nu = \frac{p}{r} \quad . \quad . \quad . \qquad (1)$$

$$t = \mathrm{T}\, p^{\frac{3}{2}} \quad . \quad . \quad . \qquad (2)$$

Then

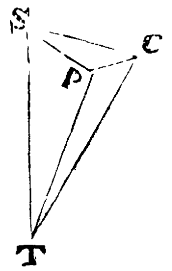
$$d\nu = \Delta d\mathrm{T} = (\text{by equat. } 2) \left(\frac{dt}{p^{\frac{3}{2}}} - \frac{3t\,dp}{2p^{\frac{5}{2}}}\right) \Delta \quad . \quad . \quad . \quad . \qquad (a)$$

by equat (1) $\log r + 2 \log \cos \tfrac{1}{2}\nu = \log p$

hence $d \log r = \sin 1'' d\nu \tan \tfrac{1}{2}\nu + \dfrac{dp}{p}$

$$\text{or } (x)\ \frac{d \log r}{\sin 1''} = d\nu \tan \tfrac{1}{2}\nu + \frac{dp}{p} \quad . \quad . \quad . \quad . \quad . \qquad (b)$$

Let S, T, C, represent the Sun, Earth, and Comet respectively, and P the projection of the Comet on the plane of the earth's orbit.



The processes to obtain the heliocentric longitudes and latitudes, are

$$\sin \mathrm{SCT} = \frac{\sin \mathrm{STC} \times \mathrm{ST}}{r}, \ \mathrm{CST} = 180^\circ - \mathrm{SCT} - \mathrm{STC}$$

$$\sin \mathrm{CSP}\ (\text{hel. lat.}) = \frac{\sin \mathrm{CTP}\ (\text{geo. lat.}) \sin \mathrm{CST}}{\sin \mathrm{STC}}$$

$$\cos \mathrm{TSP} = \frac{\cos \mathrm{CST}}{\cos \text{hel. lat.}}$$

Comets hel. long. = Earth's hel. long. + TSP

Having thus obtained β, β' and π, π'

According to Laplace's method of computation, using an auxiliary angle A,

$$\sin^2 A = \cos^2 \tfrac{1}{2} (\beta' - \beta) \cos \pi \cos \pi'$$

$$\sin \tfrac{1}{2} V = \cos (\tfrac{1}{2} \pi + \tfrac{1}{2} \pi' + A) \cos (\tfrac{1}{2} \pi + \tfrac{1}{2} \pi' - A)$$

From these formulæ, we may proceed to investigate dV

log sin hel. lat. = log sin geo. lat. — log sin STC + log sin CST

therefore, d log sin hel. lat. = d log sin CST = d CST cot CST

But, log sin SCT = log sin STC + log ST—log r

d log sin SCT = — d log r

$$\text{or d SCT} = \frac{-\tan \text{SCT d} \log r}{\sin 1''} = -x \tan \text{SCT}$$

therefore d CST = — d SCT = x tan SCT

Hence d hel. lat. (π) = x tan SCT cot CST tan π . . . . (c)

d hel. long. (β) = d TSP

log cos TSP = log cos CST—log cos π

hence d TSP = cot TSP tan CST d CST—cot TSP tan π dπ

or substituting from equation (c) and the equation preceding (c)

d hel.long. (β)=cot TSP tan SCT x (tan CST—cot CST $\tan^2 \pi$)....(e)

log sin A = log cos $\tfrac{1}{2}(\beta' - \beta)$ + $\tfrac{1}{2}$ log cos π + $\tfrac{1}{2}$ log cos π'

Hence $2dA = -\tan A(d\beta' - d\beta)\tan\tfrac{1}{2}(\beta' - \beta) + d\pi \tan \pi + d\pi' \tan \pi$ (f)

$$\left.\begin{aligned} \text{and } dV = & -\tfrac{1}{2}(d\pi + d\pi' + 2dA) \tan \tfrac{1}{2} V \tan (\tfrac{1}{2}\pi + \tfrac{1}{2}\pi' + A) \\ & -\tfrac{1}{2}(d\pi + d\pi' - 2dA) \tan \tfrac{1}{2} V \tan (\tfrac{1}{2}\pi + \tfrac{1}{2}\pi' - A) \end{aligned}\right\} \quad (g)$$

Thus by equation (a) we obtain the value of $d\nu' - d\nu = dU$ and by equations (a), (b) (c) (e), (f) and (g), the value of dV and thence the equation U + dU=V+dV in terms containing only known quantities and the two unknown quantities dp and dt. And in a similar manner the equation

U′ + dU′ = V+ dV in terms containing only known quantities and the two unknown quantities dp and dt.

The solution of these equations furnishes the values of dp and dt.

The coefficients of the unknown quantities in these equations would be correct, if we had the elements correct; but, as the elements are only approximate, so these coefficients are only approximate, and their degrees of accuracy depend on the influence of the errors of the elements on these coefficients.

Thus, if in computing dπ, SCT should be nearly a right angle, the corresponding error of dπ would be great, compared to what it would be, if SCT were nearly 45°—In the former case, therefore, the elements would require to be much more correct than in the latter—If SCT should be so near a right angle, that it was uncertain whether it were actually greater or less than a right angle, that observation could not be used for the correction of the elements; and so in similar cases. The magnitude of the angles SCT, CST, and TSP, will therefore point out the fitness of the observations used, and the degree of exactness required in the approximate elements.

When Elliptic Elements are required, equations (*a*) and (*b*) are to be adapted to an elliptic orbit. Then another unknown quantity viz., *dm*, *m* being = axis major,
will be introduced; and so four observations at least must be used, as there will be three equations with three unknown quantities.

Application to the Comet of July 1819.

The observations made on the north meridian, at the Observatory of Trinity College, Dublin, gave

1819.	Mean time of observation at Observatory T. C. D.	Longitude by observation.	Latitude by observation. N.
July 4	12^h 7′ 21″, 8	100 37 35, 7	22° 14 53, 9
5	12 7 38, 4	101 16 32, 3	23 33 1, 5
6	17 7 46, 4	101 54 0, 0	24 39 30, 4

The latitudes and longitudes are corrected* for aberration and parallax, and the longitudes are reckoned from the mean equinox.

The method of Laplace† is very convenient for the first approximation, when the observations are near together, and made with such exact meridian instruments as we possess at our Observatory.

That method gave me, from the above longitudes and latitudes,‡ perihelion dist. $=$,3397, and time of perihelion June 27^d. 15^h. 5′. 28″.

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* A first approximation, which I computed merely to obtain the outlines of the orbit, enabled me to obtain the corrections for aberration and parallax.

† Mec. Cel. tom. 1. p. 221, &c,

‡ The sun's places were computed from the Nautical Almanac, but the log. of the sun's distance was taken from the Conn. des Temps, the log. of the distances in the Nautical Almanac not being sufficiently exact to be used in the correction of the approximate elements.

By observations on the north meridian, July 4, 13, and 20, we have

1819.	Mean time of observation at Observatory T. C. D.	Longitude by observation.	Latitude by observation. N.
July 4	12h 7′ 21, 8	100 37 35, 7	22° 14 53, 9
13	12 4 29, 0	105 41 6, 4	28 52 5, 3
20	11 55 13, 3	108 43 50, 1	30 9 58, 5

The observations of July 4 and 13, and the above approximate elements, give U—V = —588″,7.

And those of July 4, and July 20, give U′—V′ = —655, 5.

The processes for finding U, U′ and V, V′ give the angles, by which the coeff. of dp and dt are determined. By help thereof we find

$$d\nu = \overset{\prime\prime}{18841}\, dt - \overset{\prime\prime}{572072}\, dp$$

$$d\nu' = 9380\, dt - 657490\, dp$$

$$d\nu'' = 5859\, dt - 595680\, dp$$

$$x = \frac{d \log r}{\sin 1''} = 7555\, dt + 377800\, dp$$

$$x' = \frac{d \log r'}{\sin 1''} = 7531\, dt + 79355\, dp$$

$$x'' = \frac{d \log r''}{\sin 1''} = 6065\, dt - 9430\, dp$$

$$d\varpi = -4{,}682x : \quad d\beta = - \; 0{,}927x$$

$$d\varpi' = +0{,}429x' : \quad d\beta' = + \; 11{,}886x'$$

$$d\varpi'' = -0{,}6412'' : \quad d\beta'' = + \; 3{,}081x''$$

By help of these quantities we find

$$dV = 4{,}639\,x + 1{,}868x'$$
$$dV' = 4{,}641\,x + 1{,}036x''$$

And finally we obtain

$$\left.\begin{array}{l} 58576\,dt + 1986238\,dp = -588{,}7 \\ 54328\,dt + 1767198\,dp = -655{,}5 \end{array}\right\} \quad (p)$$

And we deduce

$$dp = \quad 0{,}001463$$
$$dt = -0,\ 05958$$

Hence the first corrected perihelion distance = 0,341163

And . . . . time of passage
through perihelion, . June 27^{d}. 16^{h} 31′ 16″

With these quantities we again compute

$U - V = -39''{,}5$ and $U' - V' = -28''{,}2$

Substituting these quantities on the right hand side of the equations (p); we obtain again

$$dp = -{,}000112 \text{ and } dt = +,\ 003126.$$

It is unnecessary to continue the approximations farther, and these give the perihelion distance and time of perihelion sufficiently exact.

To find the place of the node, place of perihelion, &c. it is necessary to know the hel. longitudes and latitudes according as they would result from the corrected elements now found. These are easily had by substituting in the above values of $d\pi$, $d\beta$, &c. the second values of dt and dp, and applying the results to the relative longitudes and latitudes found in the second operation for finding V and V′. The elements thus obtained from the observations of July 4, 13, and 20′, are

Passage of perihelion,
Mean time at Observatory, Trinity College Dublin } June 27^{d}. 16^{h}. 26′. 46″

Perihelion distance	**0,341051**
Longitude of node	**9ˢ. 3°. 43′. 44″**
Inclination	**80. 45. 53**
Place of Perihelion	**9.17. 5. 5**

Motion direct.

Comparison of the observations with the Elements.

1819.	Mean time of observation at Observatory T.C.D.	Longitude by observation.	Latitude by observation.	Errors of elements in	
				Long.	Lat.
July 4	12^h 7′ 21″, 8	100° 37′ 35″, 7	22° 14′ 53″, 9	—17″, 2	— 4″, 0
5	12 7 38, 4	101 16 32, 3	23 33 1, 5	—15, 6	+ 4, 2
6	12 7 46, 4	101 54 0, 0	24 39 30, 4	—15, 3	—14, 8
13	12 4 29, 0	105 41 6, 4	28 52 5, 3	+ 0, 6	— 0, 2
14	12 3 28, 0	106 9 27, 6	29 9 37, 5	+ 5, 6	— 0, 7
15	12 2 21, 3	106 37 10, 0	29 24 26, 1	— 1, 5	— 1, 1
20	11 55 13, 3	108 43 50, 1	30 9 58, 5	+18, 9	+ 3, 3
21	11 53 33, 2	109 7 14, 4	30 15 7, 7	+20, 8	+ 8, 2
24	11 48 5, 4	110 14 24, 1	30 25 52, 5	+16, 9	+14, 9
28	11 39 57, 9	111 36 44, 9	30 33 3, 9	+20, 9	+13, 9